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Use of Ion Probes in Supersonic Plasma Flow

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In this paper, we shall discuss experimental results obtained using electrostatic probes that were biased to collect saturated ion currents. The probes have been used to determine the freestream charge density and the ionization rise time in the supersonic flows generated by an arc-heated, pressure-driven shock tube. Freestream charge density was measured in the plasma produced by a magnetically driven shock tube. The charge density varied from 10^{10} to 10^{15} electrons/cm^3 and the initial shock tube pressure varied from 0.02 to 1 torr. The measurements of ionization rise time agree with other results at Mach numbers where comparison is possible. A region of Mach numbers not previously covered has also been studied so that complete data on the ionization rise time as a function of Mach number for air now exists for Mach numbers from $M = 8$ to $\approx = 26$. The value of freestream charge density has been inferred from the measured ion currents to cylindrical wires and wedge-shaped electrodes using a theory described below. The agreement between the inferred charge density and the values obtained from microwave interferometers and equilibrium calculations are within a factor of 2. At pressures for which shocks form around the cylindrical wires, making probe interpretation difficult, a new type of probe—a small half-angle wedge—has been calibrated. It is shown that a simple theory can account for the measured results. A more refined theory is required if this probe is to be used without calibration.

Nomenclature

- n_∞ = freestream charge density (it is assumed that the electron and ion densities are equal in the freestream)
- v_{th} = freestream thermal velocity of ions = $(8kT/\pi M_+)^{1/2}$
- η = eV_p/kT
- V_p = probe potential
- T = ion temperature; when electron temperature > ion temperature use electron temperature
- k = Boltzmann constant
- M_+ = ion mass
- γ = a/r_p
- a = sheath radius
- r_p = probe radius
- S = speed ratio = v_f/v_+
- v_f = flow velocity
- v_+ = $(2kT/M_+)^{1/2}$
- L = probe length
- e = charge of positive ion

Introduction

THE measurement of charge density levels and the spatial and temporial variation is important in many areas of plasma physics. The use of electrostatic probes to measure these quantities in supersonic plasma flows is discussed both theoretically and experimentally.

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Theory

A. Free Molecular Cylindrical Probes

When a cylindrical conductor that is small compared to the mean free path is inserted into a plasma, and a voltage large enough to repel all the electrons is applied between the cylinder and another (larger area) electrode in the plasma, the cylinder will collect positive ion current, and a positive ion sheath will form around the probe. In general, the current will be given by an expression of the form

$$I = (n_\infty e v_{th}/4)F(\eta,\gamma,S)2\pi r_p L \tag{1}$$

1. *Qualitative relationships*

$\gamma \approx 1$

If the sheath is thin the current will be closely approximated by

$$I = (n_\infty e v_{th}/4)(2\pi r_p L) \tag{2}$$

where the first term in parenthesis is the random charge flux density and the second term is the physical area of the probe. If the plasma is flowing, there will be, in addition, a convective term of the form

$$I_c \approx (n_\infty e v_f)(2r_p L) \tag{3}$$

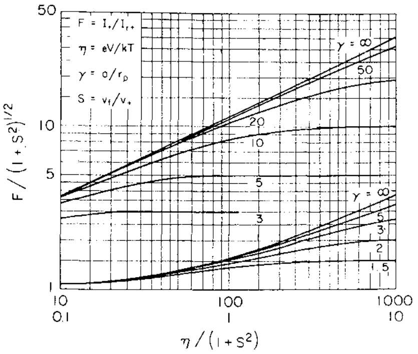

Fig. 1 Normalized probe current as a function of η, with a/r_p as a parameter.

where the first term is the convection charge flux density and the second term is the projected area that will collect this flux.

If the ratio of v_f/v_{th} ($\sim$Mach number) is much less than 1, the convective term is negligible and the current is proportional to the thermal velocity and hence $T^{1/2}$. If $v_f/v_{th} \gg 1$ the convective term dominates and the current is proportional to the flow velocity and independent of temperature. In both cases, the current is independent of probe potential. The transition from subsonic to supersonic flow is treated below.

$\gamma > 1$

When the sheath radius is significantly larger than the physical radius, it is possible for the probe to collect a current associated with the sheath or absorption radius (whichever is smaller), rather than the probe radius. The absorption radius is a function of the probe potential and the energy with which charges enter the sheath.[1]

$$r_a = r_p[1 + (V_p/V_0)]^{1/2} \quad (4)$$

where r_a = absorption radius and eV_0 = initial energy of charges.

For a stationary plasma, $V_0 = kT/e$ and $r_a/r_p = (1 + \eta)^{1/2}$. Furthermore, if $\eta \gg 1$, $r_a/r_p \approx \eta^{1/2}$. Then if r_a is less than the sheath radius, the current will be proportional to r_a (*i.e.*, if $\eta^{1/2} < \nu$, then $I \propto \eta^{1/2}$) and hence increase with $V_p^{1/2}$. If the sheath radius is smaller than r_a the current will be proportional to the sheath radius (i.e., if $\gamma < \eta^{1/2}$, then $I \propto \gamma$).

For flowing plasmas, V_0 is equal to the sum of the thermal and convective energies.

$$V_0 = [kT + (1/2)M_+v_f^2]/e = (kT/e) \times [1 + (v_f^2/v_+^2)] \quad (5)$$

Thus, the effect of increasing flow velocity is to decrease the absorption radius. For high Mach numbers, the absorption radius is inversely proportional to the Mach number. This effect alone would decrease the current collected. However, the introduction of convection also increases the flux density crossing the absorption radius. For high Mach numbers, this increase is directly proportional to the Mach number. Thus, the decreasing absorption radius and increasing flux density cancel each other, so that the current collected at high Mach numbers, when the sheath radius is large, is the same as it is with no flow velocity.

When the sheath is small, the absorption radius is larger than the sheath radius and thus, the sheath radius determines the current collected. Then the effect of convection is only to increase the flux density into the sheath. Thus, the largest effects of convection are seen when the sheath is small.

2. *Quantitative results*

The results that have been discussed above have been derived by Smetana.[3] In comparing his results to the analysis for stationary plasmas, it was found that the stationary plasma solutions gave identical results if certain normalizing parameters were modified to include the flow effects.

Figure 1 is a plot of the factor F, by which the current collected (I_+) is larger than the random flux to the physical area of the probe (I_{r+}) as a function of the ratio of potential to kinetic energy. This figure was derived by Hok[2] et al. from a Longmuir-type analysis for a stationary plasma. It exhibits the usual Langmuir-probe ion-saturation phenomena. When $\eta^{1/2} \gg \gamma$, $F = \gamma$, and the current is limited by the sheath size. Where $\gamma \gg \eta^{1/2}$, $F = (2/\pi^{1/2})(\eta + 1)^{1/2}$ and the current is limited by the potential. The coordinates of Hok's analysis were transformed as shown in Fig. 1 to include the effects of flow. When transformed in this manner, the results agree quite well with Smetana's calculation.[3] F is now the factor by which the current is larger than the total flux (random plus convective) into the physical area of the probe. The abscissa is still the ratio of the potential energy to the kinetic energy including convection $eV_p/(kT + \frac{1}{2}Mv_f^2)$.

An estimate of sheath size is necessary to use Fig. 1. This has been obtained by assuming that the probe and its surrounding sheath behave as a coaxial diode, with the edge of the sheath acting as a cathode, emitting positive ions, and the probe as the anode, collecting the ions.[4]

Using the space-charge-limited-diode equation, a relation can be found between γ and the measured current, potential, and probe size. This is plotted in Fig. 2. This will be only a rough estimate for supersonic flows, but since the absorption radius decreases with increasing Mach number the effect of the sheath tends to be diminished. If $\eta/S^2 \gg 1$ and $\gamma > \eta^{1/2}/S$, then the current collected on a probe with a large sheath is the same as if the plasma were stationary.

In order to find the charge density from a measurement of ion-saturation current, one would compute the factor that is the abscissa in Fig. 2. This involves the probe size, potential, and collected current. The probe floating potential is approximately 5 kT/e. Therefore, the value of V_p may be approximated by the sum of the applied voltage plus 5 kT/e. Entering Fig. 2 at the proper abscissa, we find γ. One can then determine F from Fig. 1, if the speed ratio S and the temperature are known. When F is known, the freestream charge density can be calculated from Eq. (1). Experimental confirmation of this theory and its application to regimes not strictly free molecular will be discussed below.

B. Wedge-Shaped Probes

It is often impossible to make a probe physically strong enough to withstand the plasma environment and still be smaller than the mean free path. When the mean free path becomes much smaller than the probe radius, a shock will form around the probe. The relation between the current measured by the probe and the incident charge

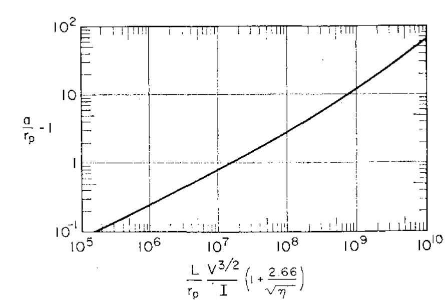

Fig. 2 a/r_p as a function of the measured parameters.

density then becomes dependent upon an analysis of the ionization and recombination processes that are important in the stagnation region. This can become a formidable problem and often depends upon a knowledge of the very properties that the probe is being used to measure.

One way to minimize the effect of additional ionization processes is to make the probe a narrow wedge, as shown in Fig. 3. Under these conditions the wedge structure may be large compared to the mean free path, but because an attached oblique shock is formed, the properties across the shock are not as strongly different as when a stagnation region is formed. Further, there is no stagnation region, so that the gases can flow past the wedge with almost their incident velocity. If the probe is made short, the time to flow over the probe can be made short compared to the ionization relaxation time, thus eliminating this factor from the analysis.

If these conditions hold, the effect of the oblique shock is to slightly compress and heat the incident plasma. These factors are shown in Fig. 4 for several wedge half angles. When ionization effects due to the probe are not important, these probes may be calibrated under particular conditions of charge and gas density, temperature, and flow velocity without regard to the degree of nonequilibrium in the gas. We have calibrated these probes against microwave interferometers and equilibrium charge density calculations. These results will be shown below. Since the probe current was shown to be linearly related to the freestream charge density, we were able to use them to measure the ionization rise time at pressures for which free molecular cylinders were difficult to fabricate. The wedge probes had spatial resolution superior to the microwave interferometers.

We did not carry out a thorough analysis of the current to the wedge in terms of the sheath structure and flowfield behind the oblique shock. But, for purposes of comparison to the random flux, we computed the current, assuming frozen flow and that the wedge collected the random flux behind the oblique shock. The results were

$$I = (n_2 e v_{th2}/4)\rho_3/\rho_2(T_3/T_2)^{1/2}A_3 \qquad (6)$$

where ρ_3/ρ_2 and T_3/T_2 are the gas density and temperature ratios across the oblique shock and A_3 is the probe area. From this equation, the incident freestream charge density n_2 can be computed. As will be seen below, this value agrees well with the actual value of freestream charge density, which implies that the probe was collecting the random flux in region 3. This shows that over the range of measurements described below the above probe theory is a suitable model. For the purposes of the results on ionization rise time, it is sufficient that we obtained a calibration current as a function of freestream charge density with this probe.

C. Electron Temperature Considerations

Throughout this paper, we have not distinguished between electron, ion, and gas temperature. For probe operation in the saturated ion current mode, the most important temperature is the electron temperature, so long as it is equal to or greater than the ion temperature. For stationary plasmas if the electron temperature is used in the expressions given above (effectively assuming $T_i/T_e \simeq 0$), the maximum error that results is about 60%. For supersonic flow conditions (supersonic with respect to the ion velocity) in which convection dominates, the effect of electron temperature becomes even less important. When the sheath is thin, the current depends only on the flow velocity.

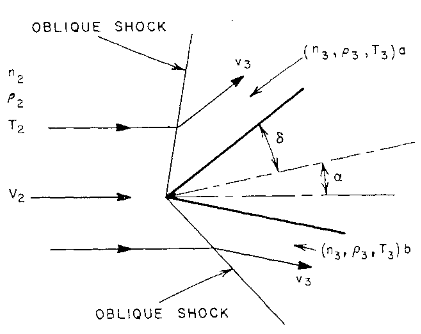

Fig. 3 Wedge with an attached oblique shock.

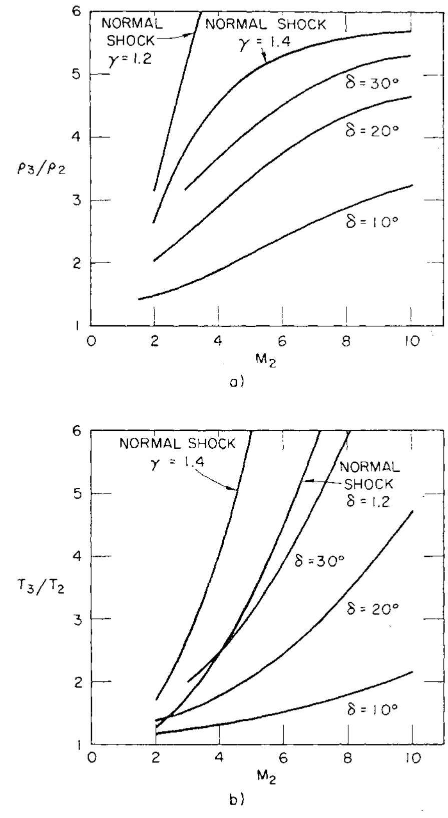

Fig. 4 Gas density and temperature jumps across an oblique shock as a function of Mach number of the incident flow.

Experimental Results

Except for one set of measurements at electron densities of 10^{15} electron/cm^3 all of the measurements were performed in a 12-in. diam arc-heated pressure-driven shock tube. The driven gas was dry air in all of the ionization rise time measurements. A liquid nitrogen cold trap was used to remove any water vapor. The gas species were measured before the shots with a quadrupole mass spectrometer. It was found that if the water vapor content was not reduced it had up to an order of magnitude effect on the equilibrium charge density at an initial shock tube pressure of 0.1 torr. With respect to cleanliness, it was found that if the water vapor was removed the charge density reached the equilibrium value and ionization rise times were the same as other measured and calculated values,[6,7] even if the tube was not cleaned for more than 30 shots.

A. Verification of Probe Theories

1. *Cylindrical probes*

Microwave interferometers operating at approximately 10 and 30 GHz were used to check the theory of cylindrical wire probes. At an initial shock tube pressure of 0.1 torr, the probe radius (5 mil) was about 2.5 neutral-neutral mean free paths (λ_{nn}), so that this probe was not operating in a strictly

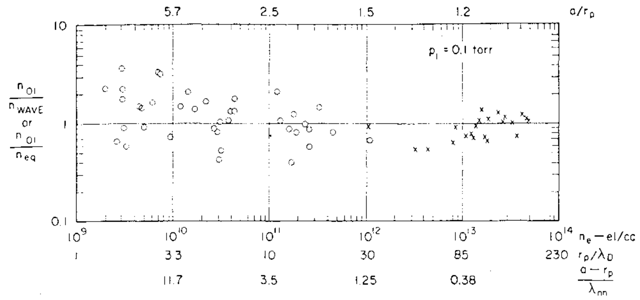

Fig. 5 Ratio of electron densities inferred from 5-mil wire probes and from microwave data as a function of electron density inferred from microwave data (p_1 = 0.1 torr).

free molecular regime. Nevertheless, a comparison was made between the microwave inferred charge density and that inferred from the cylindrical probes. Later measurements were made comparing the 5-mil cylinders ($r_p/\lambda_{nn} \approx 2.5$) to 0.5-mil cylinders ($r_p/\lambda_{nn} \approx 0.25$), which were indeed operating in a free molecular regime. The results are discussed below.

The results for the 5-mil-radius probes are shown in Fig. 5. The ordinate is the ratio of the charge density inferred from the 5-mil-radius probes to either the microwave-inferred charge density ($3 \times 10^9 < n_\infty < 10^{12}$ electron/cm^3) or the equilibrium values ($3 \times 10^{12} < n_\infty < 6 \times 10^{13}$ electron/cm^3) calculated from the measured shock speed. The abscissa is the freestream charge density. At all but the lowest charge densities the data scatters within a factor of about ± 2 about a value of 1. This shows that, within the data scatter of a factor of 2, the 5-mil-radius probe behaved like a free molecular probe interpreted as discussed above, over a range of charge densities from 10^{10} to 10^{14} electron/cm^3. The increased data scatter below 10^{10} electron/cm^3 is due to uncertainties in the microwave measurements due to the small phase shifts.

We have also plotted on the abscissa the ratio of probe radius to Debye length (λ_D), the ratio of sheath-minus-probe radius to the neutral-neutral mean free path, and the ratio of sheath to probe radius (γ). Over the range of charge densities, we go from a large sheath to a small sheath condition. Also for charge densities less than about 10^{12} electron/cm^3 there were collisions in the sheath. This data indicate that even though the probe was several neutral-neutral mean free paths in radius and had some collisions in the sheath, a free molecular theory was adequate for better than a factor of 2 accuracy.

The data show that the probe current could be interpreted to yield charge density to a factor of 2 accuracy. Further, the current was linear with freestream charge density and thus suitable for making ionization rise time measurements.

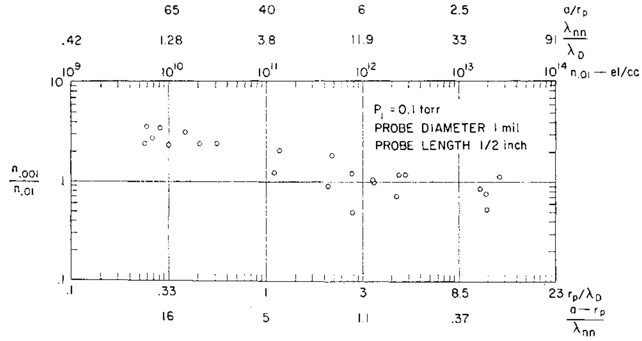

Fig. 6 Ratio of charge density inferred from 1-mil diam probe to that from the 10-mil diam probe as a function of the freestream charge density.

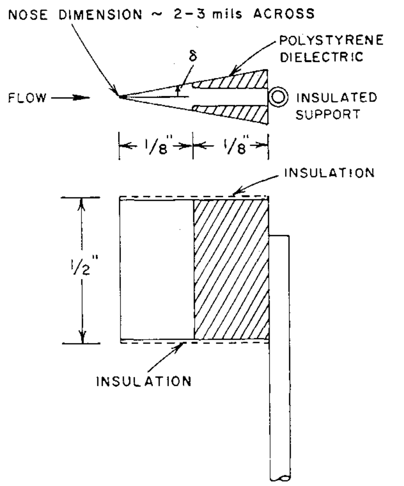

Fig. 7a Sketch of wedge probes.

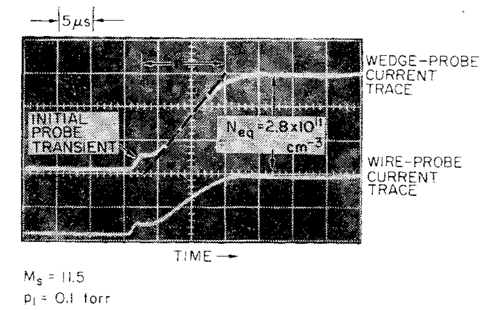

Fig. 7b Illustration of rise-time measurement using two different electrostatic probes.

In order to obtain some strictly free molecular data, a probe 0.5 mil in radius and $\frac{1}{8}$-in. long was used for measurements at an initial pressure of 0.1 torr. The results are shown in Fig. 6. The data have been plotted as the ratio of the charge density inferred from the 1-mil-diam probe to that inferred from the 10-mil-diam probe as a function of the charge density inferred from the 10-mil probe. Note that the charge density inferred from the 10-mil probe under these conditions had already been shown to agree with microwave interferometers and equilibrium calculations. Therefore the 10-mil inferred charge density is an accurate measure of the freestream charge density. Additional scales on the abscissa of Fig. 6 show the sheath-to-probe radius, mean free path to Debye length, probe radius to Debye length, and sheath thickness to mean free path.

The mean free path is about four times the probe radius, so that the probe itself is free molecular. At charge densities above 10^{12} el/cm^3, the sheath thickness is less than a mean free path, so that at these densities the probe is operating in a free molecular regime. The data show that the 1-mil- and 10-mil-diam probes agree at these densities within the data scatter of about a factor of 2. At 10^{12} electron/cm^3, the sheath-to-probe radius is about 6, and it decreases to about 2 at the highest charge densities measured. This shows that the theory is capable of taking into account large sheaths.

At densities less than 10^{12} electron/cm^3, there is a slow increase in the density inferred by the 1-mil probe as compared to the 10-mil probe. At a density of 10^{10} electron/cm^3 the ratio between the two probes is about 3. At this density, the sheath is large—65 times the probe radius and is 16 mean free paths thick. The results are consistent with the hypothesis that the absorption radius with collisions is larger than when there are no collisions. The effect of collisions appears to allow the potential field to be more effective in pulling the positive ions into the probe surface. We have no quantitative theory to account for these effects.

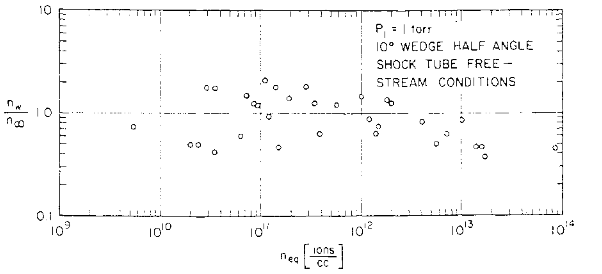

Fig. 8 Ratio of electron density inferred from 10° half-angle wedge probe to that inferred from 10-mil wire probe as a function of the electron density inferred from 10-mil wire probe (p_1 = 1.0 torr).

2. *Wedge Probes*

Measurements were made using a wedge probe with a 10° half-angle and dimensions shown in Fig. 7a. Figure 7b shows a typical oscilloscope trace made with a wedge probe and with a cylindrical wire probe illustrating how the rise time data was measured. The initial rise during the first couple of μsec is a current transient, seen at these and lower charge density levels, that is related to the approach of the ionized front through the d.c. electrostatic field between the probes and the tube wall. It can be reduced considerably by covering the probe feed line with a coaxial conductor.

The results for an initial shock tube pressure of 1 torr are shown in Fig. 8. The ordinate is the ratio of charge density inferred from the wedge probe current (using the theory discussed above) to the equilibrium charge density based on the measured shock velocity. For charge densities between 10^{10} and 10^{14} electron/cm^3 the wedge probe indicated equilibrium values plus or minus a factor of about 2. Similar results were obtained at an initial pressure of 0.1 torr. These results give some credence to the probe interpretation discussed above, but care should be taken in using this theory in other flows, since the effects of a number of parameters (e.g., Mach number, gas density) are not known. The important point for the ionization rise time studies is that the inferred charge density is linearly related to the freestream density.

B. Charge Density Measurements

The results of measurements of equilibrium charge density using wedge probes at 1 torr and 10-mil-diam cylinders at 0.1 and 0.05 torr are shown in Fig. 9. The charge density inferred from the probes agrees with the equilibrium values to better than a factor of 2 up to a shock speed of 9 mm/μsec. In all of these measurements, a liquid-nitrogen cold trap was used to remove water vapor, but the tube was not cleaned for many shots. We conclude that the requirements on cleanliness in order to obtain predicted charge densities in shock tubes are not nearly so severe as many experimenters have assumed and as may indeed be required for studying other phenomena—dissociation, recombination, radiation, etc.

C. Ionization Rise Time Measurements

The probes were used to measure the ionization rise time in air over a range of pressures. The rise time τ is defined by

$$\tau = n_{pk}/(dn/dt)_{\max} \tag{7}$$

where n_{pk} is the peak charge density. When τ was short, a low pressure was used to give sufficient resolution in time. When the Mach number was low, so that $p\tau$ was high, a higher pressure was used to make τ short compared to the test time. The measured results are shown in Fig. 10.

It can be seen that these results not only fit the calculated values of Thompson,[5] but also agree with the measured

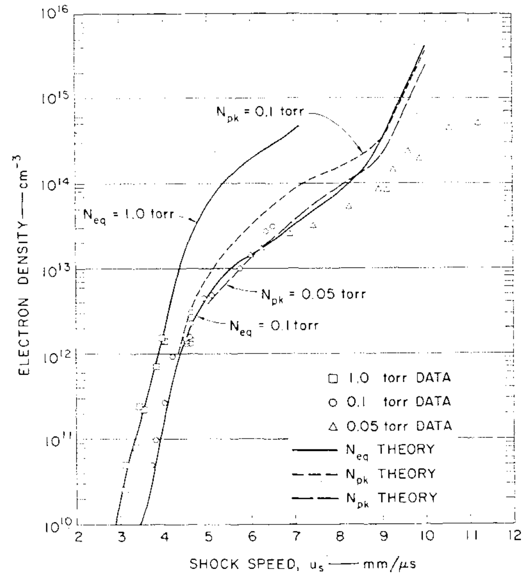

Fig. 9 Measured electron density as a function of shock speed, compared with predicted values.

results of Frohn and deBoer[6] at the low-speed end and of Lin and Teare[7] at the high-speed end. Our results also fill the gaps in the data between the previous experiments. Thompson's calculations assumed a recombination rate 3 times faster than Lin's nominal value. Lin's calculation for a recombination rate 3 times faster than his nominal value agrees quite well with our data. Our results, as well as Lin's measurements, confirm that the faster recombination rate is correct. A recent note by Hansen gives theoretical grounds for the faster rate.[8]

At very high Mach numbers (above M = 25) Wilson has made measurements that show an increase and then a decrease in ionization rise time.[9] He attributes this to a change in the ionization process, from $N + O \rightarrow NO^+ + e$ to a process in which this reaction is the rate-limiting reaction that ultimately produces N^+. Finally, the decrease in τ is due to direct-electron-impact ionization.

We do not have much data at these Mach numbers. There does seem to be an increase of τ, but our measurements do not show as large an increase as Wilson saw. This is also the shock speed range where our charge density measurements did not indicate predicted values. Nevertheless, over the range of Mach numbers from M = 8 to M = 25 our measurements agree both with calculations and measurements using other sensors, where comparisons are possible.

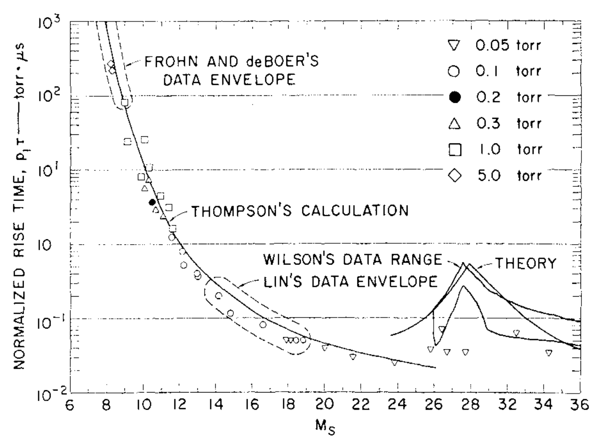

Fig. 10 Normalized ionization rise time as a function of shock Mach number for air.

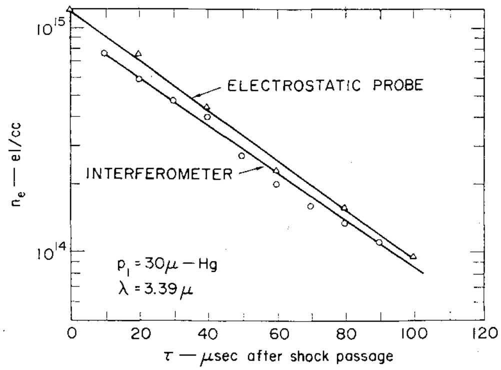

Fig. 11 Comparison of charge density inferred from a laser interferometer and an electrostatic probe.

D. Probe Experiments with a Magnetically Driven Shock Tube

In order to check probe operation at very high charge densities ($\sim 10^{15}$ electron/cm^3), measurements were made with cylindrical probe perpendicular to the flowing plasma produced in an electromagnetically driven shock tube.

This type of shock tube is a convenient source of high-electron-density plasma. A capacitor discharge across a pair of electrodes produces a plasma with currents passing through it sufficiently large to produce a pinch. The pinch produces a shock that travels down the tube. No diaphragm is used in this type of shock tube, so that the firing rate is very rapid (as high as one shot/min), compared to a conventional shock tube. However, since none of the plasma properties are predictable from shock velocity measurements, it is necessary to measure the properties.

A laser interferometer operating at wavelengths of 0.63 and 3.39 μ was used to independently measure the charge density.[10] The charge density inferred from the interferometer data as a function of time after the shock passage is shown in Fig. 11. We have also plotted the charge density inferred from a 10-mil-diam ion probe mounted perpendicular to the flow assuming the flow velocity was equal to the shock speed. The agreement between the two methods is better than a factor of 2.

Summary and Conclusions

We have shown that electrostatic probes when operated to collect saturation ion currents can be used in shock tubes to measure equilibrium charge density and follow time-varying charge density to infer quantities such as ionization rise times. Probe operation has been demonstrated in supersonic flows (Mach number ≈ 3) over a charge density range of approximately 10^{10} to 10^{15} electron/cm.[3] Using relatively simple theories, accuracy to better than a factor of 2 has been shown.

It has also been shown that free-molecular probe theory can be used even when the probe radius is 2.5 neutral-neutral mean free paths.

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